Universal Elastic Flexural Design Technique For Reinforced Masonry

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INTRODUCTION

Reinforced masonry is a structural system that evolved because of the need for masonry to resist greater forces than plain, unreinforced masonry could safely and reliably resist. The introduction of reinforcing steel bars into masonry walls changed the image of masonry from a "rule-of-thumb" design to an "engineered structural system."

Through research and testing, the physical characteristics of reinforced grouted masonry have been and are being established. The principles of engineering design have been known and classical formulas developed that can predict the performance of an engineered structural material.

BASIC ASSUMPTIONS

Reinforced masonry has used the general principles of reinforced concrete. The assumptions for the elastic working stress design of reinforced concrete apply to reinforced masonry, for both materials use steel to resist the tension forces and concrete or masonry to resist the compression forces.

The classical assumptions for the design of a heterogeneous material such as reinforced masonry or concrete are as follows:

a. Plane sections remain plane after bending.
b. Stress is proportional to strain, which is proportionial to the distance from the neutral axis.
c. Modulus of elasticity is constant throughout the member.
d. Masonry does not resist tension forces.
e. Masonry units, mortar and grout combine to be a homogeneous and isotropic material.
f. Span of the member is large compared to the depth.
g. The structural element is straight and of uniform cross section.
h. Steel is stressed equally about the center of gravity of the bars.
i. External and internal moments and forces are in equilibrium.

FLEXURAL DESIGN

A wall subjected to a flexural moment, i.e. cantilever retaining wall, will put the masonry in compression and the steel in tension.

The flexural design of reinforced masonry has been basically a condition of assumption of all properties and requirements and checking to assure the assumptions are within satisfactory limits. This is a trial-and-error method which could require several trials before a satisfactory and economical design is achieved.

Another technique has been to use published tables based on established properties, such as modular ratio, strength f'm, allowable stresses, etc. These are very convenient and have aided design. They are limited to masonry sytems that conform to the stated properties of modular ratio, allowable stresses, etc.

The current requirements in the 1985 Uniform Building Code Chapter 24 have changed the modulus of elasticity of steel, changed the modular ratio, changed the design parameters for uninspected masonry, etc. These changes have altered the design techniques for reinforced masonry.

UNIVERSAL ELASTIC FLEXURAL DESIGN

The universal elastic flexural design technique for reinforced masonry given in this paper provides a method to accommodate any modulus of elasticity, any modular ratio, any stress in masonry and steel, and provides a method to design a system without unnecessary assumptions.

Part of the technique has been used by many, but it has been extended in this paper to be all-encompassing and to be a design method rather than just an assumption and checking process.

The key to this universal technique is the standard "np" table that lists the values k, j, and 2/jk, which are obtained from the classical equation

$$k = \sqrt{(np)^2 + 2np} - np$$
$$j = 1 - k/3.$$

The "np" table has been extended to include an npj value.

The use of the 2/jk and npj values is a technique that allows the engineer to design a masonry system. It allows design and obtains the minimum steel area to satisfy requirements.

FLEXURAL DESIGN EQUATIONS

Moment based on allowable flexural compressive masonry stress (F_b)

$$M = bd^2\,(jk/2)\,F_b$$

Solve for 2/jk

$$\frac{2}{jk} = \frac{bd^2\,F_b}{M}$$

Moment based on the allowable tensile steel stresses (F_s)

$$M = bd^2\,(pj)\,F_s$$

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Multiply both sides by n and solve for npj

$$npj = \frac{nM}{bd^2 F_s}$$

Solving the equations for 2/jk and npj, the np value can be obtained from the "np" table and the steel ratio calculated:

$$p = \frac{np}{n}$$

The area of steel can then be determined

$$A_s = pbd$$

DESIGN EXAMPLES

Example No. 1, Steel Requirement.
Determine the reinforcing steel required in a masonry parapet.

Given: A 7′-high parapet wall located in earthquake zone 4; determine the reinforcing steel required.
Assume: 8″ CMU grouted at steel only w/inspection
f'm = 1,500 psi (Table No. 24-D 1985 U.B.C.)
Allowable $F_b = \frac{1}{3} \times 1{,}500$ psi
= 500 psi
d = 7.62″/2 = 3.81″ (steel in center of wall)
Assume weight of wall = 60 psf
Grade 60 steel, $f_y = 60{,}000$ psi, $F_s = 24{,}000$ psi
Allowable stresses may be increased ⅓ when moment is due to wind or earthquake
Special inspection provided

Lateral load on parapet,
$F_p = 0.8W$ (Table No. 23-J 1985 U.B.C.)
$F_p = 0.8W\,(60) = 48$ psf

Moment at connection of parapet at roof line

$$M = \frac{F_p h^2}{2} = \frac{48 \times 7^2}{2}$$
$$= 1176 \text{ ft. lbs./ft.}$$

$$\text{Modular ratio, } n = \frac{E_s}{E_m} = \frac{E_s}{1{,}000 \text{ f'm}}$$

$$= \frac{29{,}000{,}000}{1{,}000 \times 1{,}500} = 19.3$$

Determination of flexural coefficients

$$2/jk = \frac{F_b bd^2}{M} = \frac{500 \times 1.33 \times 12 \times 3.81^2}{1176 \times 12}$$

$$= 8.229$$

$$npj = \frac{nM}{bd^2 F_s} = \frac{19.3 \times 1176 \times 12}{12 \times 3.81^2 \times 24{,}000 \times 1.33}$$

$$= 0.04886$$

From the "np" table for

2/jk = 8.229 read np = 0.048
npj = 0.04886 read np = 0.054

Use the largest np value and determine the steel ratio, steel stress governs,

$$p = \frac{np}{n} = \frac{0.054}{19.3} = 0.0028$$

Note: Design using masonry stress only would result in not providing enough steel and it would be overstressed.

Area of steel:

As = pbd
= 0.0028 x 12 x 3.81
= 0.13 sq. in./ft.
Use #5 @ 24″ o.c.

Example No. 2, Moment Capacity.
Determine the moment capacity of a 10″-thick grouted brick wall that is reinforced with #6 bars at 16″ o.c. with Grade 60 steel. Steel is in center of wall and brick is Grade SW. The modulus of elasticity for the masonry was determined by test to be 1,350,000 psi. No special inspection was provided. The 1985 U.B.C. applies.

Assume: f'm = 1,800 psi

Allowable $F_b = \frac{1}{2} \times \frac{1}{3} \times 1{,}800 = 300$ psi
$F_s = 24{,}000$ psi; d = 5″

$$n = \frac{E_s}{E_m} = \frac{29{,}000{,}000}{1{,}350{,}000} = 21.5$$

$$p = \frac{A_s}{bd} = \frac{0.44}{16 \times 5} = 0.0055$$

$$np = 21.5 \times 0.0055 = 0.118$$

From the "np" table for

np = 0.118 read 2/jk = 6.001; j = 0.873

Moment capacity based on masonry stress

$$M_m = \frac{F_b bd^2}{2/jk} = \frac{300 \times 12 \times 5^2}{6.001}$$

= 15,000 in. lbs./ft.
= 1,250 ft. lbs./ft.

Moment capacity based on steel stress

$$M_s = F_s bd^2(pj) = 24{,}000 \times 12 \times 5^2\,(0.0055 \times 0.873)$$
= 34,000 ins. lbs./ft.
= 2,880 ft. lbs./ft.

Moment capacity of section is the lesser value of M_m and M_s

Masonry governs = 1,250 ft. lbs./ft.

Example No. 3, Stresses in Masonry and Steel.
Determine if the stresses in the masonry beam are acceptable. Special inspection was provided. A concrete masonry spandrel beam over a wide opening is subjected to a moment of 30,000 ft. lbs. due to live and dead load. The beam is 10″ CMU, total depth 40″ and d distance from compression edge to steel 34 inches. The beam is reinforced with two #6 bars Grade 40, f'm = 2,000 psi. Modulus of elasticity of masonry was determined to be E_m = 550 f'm.

Solution:

$$\text{Modular ratio } n = \frac{E_s}{E_m} = \frac{29{,}000{,}000}{550 \times 2000} = 26.4$$

$$\text{Steel ratio } p = \frac{A_s}{bd} = \frac{2 \times 0.44}{9.63 \times 34} = 0.0027$$

$$np = 26.4 \times 0.0027 = 0.0713$$

From the "np" table for np = 0.071

read 2/jk = 7.145 and j = 0.896

Solve for actual stresses in masonry and steel

Masonry stress

$$f_b = \frac{M}{bd^2}(2/jk) = \frac{30{,}000 \times 12}{9.63 \times 34^2}(7.145)$$

$$= 231 \text{ psi} < 667 \text{ psi} = 1/3 \times 2{,}000$$

Steel stress

$$f_s = \frac{M}{bd^2}\frac{1}{pj} = \frac{30{,}000 \times 12}{9.63 \times 34^2}\;\frac{1}{0.0027 \times 0.896}$$

$$= 13{,}370 \text{ psi} < 20{,}000 \text{ psi}$$

Stresses are acceptable

Example No. 4, Thickness of Masonry Retaining Wall and Steel Requirement
Determine the thickness of the hollow brick masonry unit and reinforcing requirement for a retaining wall that is 6′ high, retaining drainable soil at a slope of 2:1. No special inspection is provided; f'm = 2,600 psi; wall is solid grouted, f_y = 40,000 psi. E_m = 1,800,000 psi.

Solution:

Maximum allowable masonry flexural compressive stress
$F_b = \frac{1}{2}(\frac{1}{3} f'm) = 433$ psi
Maximum allowable steel flexural tensile stress
$F_s = 20{,}000$ psi

$$\text{Modular ratio, } n = \frac{E_m}{E_s} = \frac{29{,}000{,}000}{1{,}800{,}000} = 16.1$$

Depth of stress block based on allowable stresses, balanced stress design

$$k = \frac{nF_b}{F_s + nF_b}$$

$$= \frac{16.1\,(433)}{20{,}000 + 16.1\,(433)}$$

$$= 0.258$$

$$j = 1 - (k/3) = 1 - \left(\frac{0.258}{3}\right) = 0.914$$

$$2/jk = \frac{2}{0.914\,(0.258)} = 8.481$$

Equivalent fluid pressure for a 2:1 drainable backfill = 43 pcf

Moment at connection between masonry wall and concrete footing

$$M = \frac{1}{6} wh^3 = \frac{1}{6} \times 43(6)^3$$

$$= 1{,}548 \text{ ft. lbs./ft.}$$

$$\text{Moment } M_m = bd^2 F_b \left(\frac{jk}{2}\right)$$

Solve for bd^2

$$bd^2 = \frac{M}{F_b} \times 2/jk$$

$$= \frac{1548 \times 12}{433} \times 8.481$$

$$bd^2 = 364$$

$$b = 12''$$

$$d^2 = \frac{364}{12} = 30.3$$

$$d = 5.5''$$

Use 8″ hollow brick unit; d = 5.0″
Using d = 5.0″ recompute 2/jk and compute npj

Based on masonry stress

$$2/jk = \frac{bd^2 F_b}{M}$$

$$= \frac{12 \times 15^2 \times 433}{1548 \times 12}$$

$$= 6.993 \text{ (masonry)}$$

Based on steel stress

$$npj = \frac{nM}{bd^2 F_s}$$

$$= \frac{16.1 \times 1548 \times 12}{12 \times 5^2 \times 20,000}$$

$$= 0.04985 \text{ (steel)}$$

From the "np" table for

$2/jk = 6.99$ read $np = 0.076$

$npj = 0.04985$ read $np = 0.055$

Largest value of np controls

$np = 0.076$ (masonry controls)

$$\text{Steel ratio } p = \frac{np}{n} = \frac{0.076}{16.1}$$

$$= 0.0047$$

Area of steel per foot

$$A_s = pbd$$

$$= 0.0047 \times 12 \times 5$$

$$= 0.28 \text{ sq. in./ft.}$$

Use #7 @ 24″ o.c. $A_s = 0.30$ sq. in./ft.

Check actual stress in steel and masonry

$$p = \frac{A_s}{bd} = \frac{0.60}{24 \times 5}$$

$$= 0.0050$$

$$np = 16.1\,(0.0050)$$

$$= 0.081$$

From the "np" table for

$np = 0.081$ read $2/jk = 6.818$

$j = 0.890$

Calculate steel stress

$$f_s = \frac{M}{bd^2}\left(\frac{1}{pj}\right)$$

$$= \frac{1548 \times 12}{12 \times 5^2}\left(\frac{1}{0.0050 \times 0.890}\right)$$

$$= 13,900 \text{ psi} < 20,000 \text{ psi}$$

Calculate masonry stress

$$f_m = \frac{M}{bd^2}\left(2/jk\right)$$

$$= \frac{1548 \times 12}{12 \times 5^2}\left(6.818\right)$$

$$= 422 \text{ psi} < 433 \text{ psi}$$

All stresses satisfactory

Example No. 5, Interaction, P&M, Design.

Determine the steel requirement for a 9″ grouted brick wall 24′ high, f'm = 1,800 psi with a vertical axial load of 4,000 plf; lateral seismic load is 27 psf; vertical reinforcing steel is in center of wall, d = 4.5″; f_s = 24,000 psi. The modulus of elasticity of the masonry was determined to be 1,200,000 psi. All allowable stresses may be increased due to seismic forces. Special inspection provided.

Solution:

$$\text{Modular ratio, } n = \frac{E_s}{E_m} = \frac{29,000,000}{1,200,000} = 24.2$$

$$\text{Moment on wall, } M = \frac{wh^2}{8} = \frac{27 \times 24^2}{8}$$

$$= 1,944 \text{ ft. lbs./ft.}$$

Maximum allowable axial compressive stress

$$F_a = 0.2\,f'm\left[1 - \left(\frac{h'}{42t}\right)^3\right]$$

$$= 0.2\,(1,800)\left[1 - \left(\frac{24 \times 12}{42 \times 9}\right)^3\right]$$

$$= 176 \text{ psi}$$

Actual axial compressive stress (masonry = 90 psf)

Load at mid height of wall = 4,000 + (12 + 90) = 5,080 plf

$$f_a = \frac{P}{bt} = \frac{5,080}{12 \times 9} = 47 \text{ psi}$$

As the wall is subjected to both vertical and lateral seismic load design using the interaction unity equation

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \leqslant 1.33 \text{ (seismic conditions)}$$

Maximum flexural compressive stress, f_b, that will satisfy the above equation

$$f_b = \left(1.33 - \frac{f_a}{F_a}\right) F_b$$

$$= \left(1.33 - \frac{47}{176}\right) \frac{1800}{3}$$

$$= 638 \text{ psi}$$

Maximum allowable tensile stress in steel

$$F_s = 1.33 \text{ x } 24{,}000$$
$$= 32{,}000 \text{ psi}$$

Solve for 2/jk (masonry)

$$\frac{bd^2 f_b}{M} = \frac{12 \text{ x } 4.5^2 \text{ x } 638}{1944 \text{ x } 12} = 6.646$$

From the "np" table for

2/jk = 6.646 read np = 0.087

Solve for npj (steel)

$$npj = \frac{nM}{bd^2 f_s} = \frac{24.2 \text{ x } 1944 \text{ x } 12}{12 \text{ x } 4.5^2 \text{ x } 32{,}000} = 0.0726$$

From the "np" table for

npj = 0.0726 read np = 0.081 < 0.087

Masonry governs; np = 0.087

$$\text{Steel ratio } p = \frac{np}{n} = \frac{0.087}{24.2} = 0.0036$$

Area of steel required

$$A_s = pbd$$

$$= 0.0036 \text{ x } 12 \text{ x } 4.5$$

$$= 0.194 \text{ sq. in./ft.}$$

Use #7 @ 36″ o.c. (A_s = 0.20 sq. in./ft.)

or #6 @ 24″ o.c.

CONCLUSION

Through the use of determining the 2/jk and npj values, steel ratios and thus the steel area and specification for bar size and spacing can be obtained.

This technique is applicable for any modulus of elasticity of masonry and steel, for any value for masonry and steel stresses, and satisfies all conditions. It eliminates the need to assume all factors in design and then checking to determine if the assumptions are correct.

Through the use of the "np" table, all the flexural coefficients for elastic working stress design and analysis are there to determine stresses, moment capacity, steel ratios, etc.

It is universally applicable to all masonry systems, i.e. clay and concrete units, solid and hollow units, solid and partially grouted systems. It is applicable to beams, walls with just moment, or a combination of load and moment.

APPENDIX I

REFERENCES

"Concrete Masonry Design Manual" by Albyn Mackintosh, C.E. and William Wyman, AIA, published 1955 by Concrete Masonry Association.

"Masonry Design Manual" by James E. Amrhein, S.E., James J. Kesler, S.E., et al, 3rd edition, published 1979.

"Modular and Steeltyd Reinforced Grouted Brick Masonry Design" by William Taggart, S.E., published 1945 by Davidson Brick Company.

"Reinforced Masonry Engineering Handbook" by James E. Amrhein, S.E., 4th edition, published 1983 by Masonry Institute of America.

TABLE 1a
FLEXURAL COEFFICIENTS
BASED ON np VALUES

$n = \frac{E_s}{E_m}$ $\quad p = \frac{A_s}{bd}$ $\quad k = \sqrt{2np + (np)^2} - np$ $\quad 2/jk = \frac{bd^2 f_m}{M}$

$npj = \frac{nM}{bd^2 f_s}$ $\quad j = 1 - \frac{k}{3}$ $\quad f_m = \frac{M}{bd^2}\left(\frac{2}{jk}\right)$ $\quad f_s = \frac{M}{bd^2}\left(\frac{1}{pj}\right)$

np	2/jk	j	k	npj
.001	46.409	.985	.044	.000985
.002	33.319	.980	.061	.001960
.003	27.523	.975	.075	.002925
.004	24.069	.971	.086	.003884
.005	21.713	.968	.095	.004840
.006	19.975	.965	.104	.005790
.007	18.625	.963	.112	.006741
.008	17.537	.960	.119	.007680
.009	16.636	.958	.125	.008622
.010	15.875	.956	.132	.00956
.011	15.220	.954	.138	.010494
.012	14.649	.952	.143	.011424
.013	14.145	.950	.149	.012350
.014	13.697	.949	.154	.013286
.015	13.294	.947	.159	.014205
.016	12.930	.945	.164	.015120
.017	12.599	.944	.168	.016048
.018	12.296	.942	.173	.01696
.019	12.017	.941	.177	.01788
.020	11.759	.940	.181	.01880
.021	11.521	.938	.185	.01970
.022	11.298	.937	.189	.02061
.023	11.091	.936	.193	.02153
.024	10.897	.935	.196	.02244
.025	10.714	.933	.200	.02333
.026	10.543	.932	.204	.02423
.027	10.381	.931	.207	.02513
.028	10.227	.930	.210	.02604
.029	10.082	.929	.214	.02641
.030	9.945	.928	.217	.02784
.031	9.814	.927	.220	.02874
.032	9.689	.926	.223	.02963
.033	9.570	.925	.226	.03053
.034	9.456	.924	.229	.03142
.035	9.348	.923	.232	.03231
.036	9.244	.922	.235	.03319
.037	9.144	.921	.238	.03408
.038	9.048	.920	.240	.03496
.039	8.956	.919	.243	.03584
.040	8.868	.918	.246	.03672
.041	8.782	.917	.248	.03760
.042	8.700	.916	.251	.03847
.043	8.621	.916	.253	.03939
.044	8.545	.915	.256	.04026
.045	8.471	.914	.258	.04113
.046	8.399	.913	.261	.04200
.047	8.330	.912	.263	.04286
.048	8.263	.911	.266	.04373
.049	8.199	.911	.268	.04464
.050	8.136	.910	.270	.04550

np	2/jk	j	k	npj
.051	8.075	.909	.272	.04545
.052	8.016	.908	.275	.04722
.053	7.958	.908	.277	.04812
.054	7.902	.907	.279	.04898
.055	7.848	.906	.281	.04983
.056	7.795	.906	.283	.05074
.057	7.744	.905	.285	.05159
.058	7.694	.904	.287	.05243
.059	7.645	.903	.290	.05328
.060	7.598	.903	.292	.05418
.061	7.552	.902	.294	.05502
.062	7.507	.901	.296	.05586
.063	7.462	.901	.298	.05676
.064	7.419	.900	.299	.05760
.065	7.378	.900	.301	.05850
.066	7.337	.899	.303	.05933
.067	7.296	.898	.305	.06017
.068	7.257	.898	.307	.06106
.069	7.219	.897	.309	.06189
.070	7.182	.896	.311	.06272
.071	7.145	.896	.312	.06392
.072	7.109	.895	.314	.06444
.073	7.074	.895	.316	.06534
.074	7.040	.894	.318	.06616
.075	7.006	.894	.319	.06705
.076	6.973	.893	.321	.06787
.077	6.941	.892	.323	.06868
.078	6.909	.892	.325	.06958
.079	6.878	.891	.326	.07039
.080	6.848	.891	.328	.07128
.081	6.818	.890	.330	.07209
.082	6.788	.890	.331	.07298
.083	6.759	.889	.333	.07379
.084	6.731	.889	.334	.07468
.085	6.703	.888	.336	.07548
.086	6.676	.887	.338	.07628
.087	6.649	.887	.339	.07717
.088	6.623	.886	.341	.07797
.089	6.597	.886	.342	.07885
.090	6.572	.885	.344	.07965
.091	6.547	.885	.345	.08054
.092	6.522	.884	.347	.08133
.093	6.498	.884	.348	.08221
.094	6.474	.883	.350	.08300
.095	6.451	.883	.351	.08389
.096	6.428	.882	.353	.08467
.097	6.405	.882	.354	.08555
.098	6.383	.882	.355	.08644
.099	6.361	.881	.357	.08722
.100	6.340	.881	.358	.0881

TABLE 1b
FLEXURAL COEFFICIENTS
BASED ON np VALUES

$$n = \frac{E_s}{E_m} \qquad p = \frac{A_s}{bd} \qquad k = \sqrt{2np + (np)^2} - np \qquad 2/jk = \frac{bd^2 f_m}{M}$$

$$npj = \frac{nM}{bd^2 f_s} \qquad j = 1 - \frac{k}{3} \qquad f_m = \frac{M}{bd^2}\left(\frac{2}{jk}\right) \qquad f_s = \frac{M}{bd^2}\left(\frac{1}{pj}\right)$$

np	2/jk	j	k	npj
.101	6.318	.880	.360	.08888
.102	6.297	.880	.361	.08976
.103	6.277	.879	.362	.09054
.104	6.257	.879	.364	.09142
.105	6.237	.878	.365	.09219
.106	6.217	.878	.366	.09307
.107	6.197	.877	.368	.09384
.108	6.178	.877	.369	.09472
.109	6.159	.877	.370	.09559
.110	6.141	.876	.372	.09636
.111	6.122	.876	.373	.09724
.112	6.104	.875	.374	.09800
.113	6.086	.875	.376	.09888
.114	6.069	.874	.377	.09964
.115	6.051	.874	.378	.10051
.116	6.034	.874	.379	.10138
.117	6.017	.873	.381	.10214
.118	6.001	.873	.382	.10301
.119	5.984	.872	.383	.10377
.120	5.968	.872	.384	.10464
.121	5.957	.871	.386	.10539
.122	5.936	.871	.387	.10626
.123	5.920	.871	.388	.10713
.124	5.905	.870	.389	.10788
.125	5.890	.870	.390	.10875
.126	5.874	.869	.392	.10949
.127	5.860	.869	.393	.11036
.128	5.845	.869	.394	.11123
.129	5.830	.868	.395	.11197
.130	5.816	.868	.396	.11284
.131	5.802	.868	.397	.11371
.132	5.788	.867	.398	.11444
.133	5.774	.867	.400	.11531
.134	5.760	.866	.401	.11604
.135	5.747	.866	.402	.11691
.136	5.733	.866	.403	.11778
.137	5.720	.865	.404	.11851
.138	5.707	.865	.405	.11937
.139	5.694	.865	.406	.12024
.140	5.681	.864	.407	.12096
.142	5.656	.863	.410	.12255
.144	5.631	.863	.412	.12427
.146	5.607	.862	.414	.12585
.148	5.584	.861	.416	.12743
.150	5.560	.861	.418	.12915
.152	5.538	.860	.420	.13072
.154	5.516	.859	.422	.13244
.156	5.494	.859	.424	.13400
.158	5.473	.858	.426	.13556
.160	5.452	.857	.428	.13712

np	2/jk	j	k	npj
.162	5.431	.857	.430	.13883
.164	5.411	.856	.432	.14038
.166	5.392	.855	.434	.14193
.168	5.372	.855	.436	.14364
.170	5.353	.854	.437	.14518
.172	5.335	.854	.439	.14689
.174	5.316	.853	.441	.14842
.176	5.298	.852	.443	.14995
.178	5.281	.852	.445	.15166
.180	5.263	.851	.446	.15318
.182	5.246	.851	.448	.15488
.184	5.236	.850	.450	.15640
.186	5.213	.849	.452	.15791
.188	5.197	.849	.453	.15961
.190	5.181	.848	.455	.16112
.192	5.165	.848	.457	.16282
.194	5.150	.847	.458	.16432
.196	5.135	.847	.460	.16601
.198	5.120	.846	.462	.16751
.200	5.105	.846	.463	.16920
.202	5.091	.845	.465	.17069
.204	5.076	.844	.467	.17218
.206	5.062	.844	.468	.17386
.208	5.049	.843	.470	.17534
.210	5.035	.843	.471	.17703
.212	5.022	.842	.473	.17850
.214	5.008	.842	.474	.18019
.216	4.995	.841	.476	.18166
.218	4.983	.841	.477	.18334
.220	4.970	.840	.479	.18480
.222	4.957	.840	.480	.18648
.224	4.945	.839	.482	.18794
.226	4.933	.839	.483	.18961
.228	4.921	.838	.485	.19106
.230	4.909	.838	.486	.19274
.232	4.898	.837	.488	.19418
.234	4.886	.837	.489	.19536
.236	4.875	.837	.490	.19753
.238	4.864	.836	.492	.19897
.240	4.853	.836	.493	.20064
.242	4.842	.835	.495	.20207
.244	4.831	.835	.496	.20374
.246	4.821	.834	.497	.20516
.248	4.810	.834	.499	.20683
.250	4.800	.833	.500	.20825
.252	4.790	.833	.501	.20992
.254	4.780	.832	.503	.21133
.256	4.770	.832	.504	.21299
.258	4.760	.832	.505	.21466
.260	4.750	.831	.507	.21606

TABLE 1c
FLEXURAL COEFFICIENTS
BASED ON np VALUES

$n = \frac{E_s}{E_m}$ $p = \frac{A_s}{bd}$ $k = \sqrt{2np + (np)^2} - np$ $2/jk = \frac{bd^2 f_m}{M}$

$npj = \frac{nM}{bd^2 f_s}$ $j = 1 - \frac{k}{3}$ $f_m = \frac{M}{bd^2}\left(\frac{2}{j\,k}\right)$ $f_s = \frac{M}{bd^2}\left(\frac{1}{pj}\right)$

np	2/jk	j	k	npj
.262	4.741	.831	.508	.21772
.264	4.731	.830	.509	.21912
.266	4.722	.830	.510	.22078
.268	4.713	.829	.512	.22217
.270	4.704	.829	.513	.22383
.272	4.695	.829	.514	.22549
.274	4.686	.828	.515	.22687
.276	4.677	.828	.517	.22853
.278	4.668	.827	.518	.22991
.280	4.660	.827	.519	.23156
.282	4.651	.827	.520	.23321
.284	4.643	.826	.521	.23458
.286	4.634	.826	.523	.23624
.288	4.626	.825	.524	.23760
.290	4.618	.825	.525	.23925
.292	4.610	.825	.526	.24090
.294	4.602	.824	.527	.24226
.296	4.594	.824	.528	.24390
.298	4.587	.823	.530	.24525
.300	4.579	.823	.531	.24690
.305	4.560	.822	.533	.25071
.310	4.542	.821	.536	.25451
.315	4.524	.820	.539	.25830
.320	4.506	.819	.542	.26208
.325	4.489	.819	.544	.26520
.330	4.472	.818	.547	.26994
.335	4.456	.817	.549	.27370
.340	4.440	.816	.552	.27744
.345	4.425	.815	.554	.28118
.350	4.410	.814	.557	.28490
.355	4.395	.814	.559	.28897
.360	4.381	.813	.562	.29268
.365	4.367	.812	.564	.29638
.370	4.353	.811	.566	.30007
.375	4.339	.810	.569	.30375
.380	4.326	.810	.571	.30780
.385	4.313	.809	.573	.31147
.390	4.300	.808	.575	.31512
.395	4.288	.807	.578	.31877
.400	4.276	.807	.580	.32280
.405	4.264	.806	.582	.32643
.410	4.252	.805	.584	.33005
.415	4.241	.805	.586	.33408
.420	4.230	.804	.588	.33768
.425	4.219	.803	.590	.34128
.430	4.208	.803	.592	.34529
.435	4.197	.802	.594	.34887
.440	4.187	.801	.596	.35244
.445	4.177	.801	.598	.35645
.450	4.167	.800	.600	.36000

np	2/jk	j	k	npj
.455	4.157	.799	.602	.36355
.460	4.147	.799	.604	.36754
.465	4.138	.798	.606	.37107
.470	4.128	.798	.607	.37506
.475	4.119	.797	.609	.37858
.480	4.110	.796	.611	.38208
.485	4.101	.796	.613	.38606
.490	4.093	.795	.615	.38955
.495	4.084	.795	.616	.39353
.500	4.076	.794	.618	.39700
.505	4.067	.793	.620	.40047
.510	4.059	.793	.621	.40443
.515	4.051	.792	.623	.40788
.520	4.043	.792	.625	.41184
.525	4.036	.791	.626	.41528
.530	4.028	.791	.628	.41923
.535	4.020	.790	.630	.42265
.540	4.013	.790	.631	.42660
.545	4.006	.789	.633	.43001
.550	3.999	.789	.634	.43395
.555	3.992	.788	.636	.43734
.560	3.985	.788	.637	.44128
.565	3.978	.787	.639	.44466
.570	3.971	.787	.640	.44859
.575	3.964	.786	.642	.45195
.580	3.958	.786	.643	.45588
.585	3.951	.785	.645	.45923
.590	3.945	.785	.646	.46315
.595	3.939	.784	.648	.46648
.600	3.932	.784	.649	.47040
.605	3.926	.783	.650	.47372
.610	3.920	.783	.652	.47763
.615	3.914	.782	.653	.48093
.620	3.908	.782	.655	.48484
.625	3.903	.781	.656	.48818
.630	3.987	.781	.657	.49203
.635	3.891	.780	.659	.49530
.640	3.886	.780	.660	.49920
.645	3.880	.780	.661	.50310
.650	3.875	.779	.662	.50635
.655	3.869	.779	.664	.51025
.660	3.864	.778	.665	.51348
.665	3.859	.778	.666	.51737
.670	3.854	.778	.667	.52126
.675	3.849	.777	.669	.52448
.680	3.844	.777	.670	.52836
.685	3.839	.776	.671	.53156
.690	3.834	.776	.672	.53544
.695	3.829	.775	.674	.53863
.700	3.824	.775	.675	.54250